An Optimal Pricing Model for Interconnection

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Introduction

There are two main characteristics ascribed to the telecommunication industry: being a natural monopoly and having network externalities. Both attributes have made regulators to intervene, as they qualify them as market failures, trying to avoid firms' practices that could harm consumers. One of the mechanisms to reach that objective has been setting the access charge among telecommunication firms. The access charge is the price that firms wanting to get to final consumers have to pay to the owner of the facility that effectively reach those consumers. Therefore, the regulator tries to avoid monopolistic behaviors of the firm owning that essential facility.

In order to set optimal access charges, the regulator has to incorporate into his model the network externalities, particularly when the penetration rates are low. This work presents a model, based on that developed by Armstrong, Doyle & Vickers (1996), and incorporates network externalities to it.

In the next section, there is an explanation of network externalities, when it is important to include them, and how to make a model of them. Then it is presented the interconnection pricing model, with a brief explanation of the original model, how the externalities modify it, and an analysis of the results. Finally, the conclusions are stated.

Network Externalities

Externality is the benefit or cost that an economic agent faces as a result of an operation where he did not participate. In the case of a network externality, it appears when the utility of consuming a good increases as more agents consume the good as well (Katz & Shapiro 1985).

It is possible to distinguish two kinds of externalities in telecommunication markets (Squire 1973): access and usage of the network. The former is the benefit of the individuals that are connected to a network and can call others on it; they also obtain a benefit when a new individual joins the network, and they are not charged because of that. The externality associated with the usage of the network comes from the benefit that individuals get from being called. This occurs when *calling party pays* for the calls he makes. The externality of access to the network is called network externality.

Squire (1973) modeled both externalities, making the curve of demand a function of the total number of subscribers, because this number obviously affects the quantity of calls they could make. On one hand, it affects directly the quantity of calls because of the price; and, on the other hand, a change in the number of subscribers produces a change in the benefit of being connected. Consequently, there will be two kinds of curves of demand. First, the *conceptual* curve of demand, that relates price and quantity of calls given a fixed number of subscribers. Secondly, an *observed* curve of demand that relates price and quantity of calls and includes the changes that causes the variable number of subscribers. The latter curve of demand will be more elastic and it is compound of the equilibrium points of the conceptual curves of demand.

Abstract

This work presents an optimal access-pricing model, based in the work of Armstrong, Doyle & Vickers (1996), but including network externalities. In the model, there is an incumbent firm owning the essential facility, and an entrant who needs to buy access to final consumers. This situation is studied comparing first-best optimum and Ramsey pricing. The findings show that prices are lower when network externalities are included, and the importance of giving some kind of incentive to the firms which introduce new technology, so that the network can grow and the consumers can get the benefits of the network externalities.

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Noted the existence of the network externalities, the question is whether it is important or not to consider them in a model for pricing interconnection between telephone companies. Barnet & Kaserman (1998) show that if network externalities are infra marginal, unsubsidized market-determined prices yield Pareto optimal results. However, they also affirm that in case network externalities are non-zero at the margin, we face a market failure, and there could be a Pareto improvement (e.g. a subsidy).

The condition necessary for a Pareto relevant network externality is a low telephone penetration rate, because in that situation, any additional subscriber to the network increases its value. As the network grows, and fewer people are out of it (not connected), the marginal external value of additional subscribers is likely to decline (Barnet & Kaserman 1998).

The model that will be described in the following section attempts to solve the problem in setting access charges that arises with the existence of a market failure caused by a low telephone penetration rate. Before developing the model is necessary to consider how to model the network externality.

According to Mitchell & Vogelsang (1991) the network externality can be assessed from the changes in the *conceptual* demand for access when a new consumer is added. Given that the conceptual demand for access takes into account a fixed number of subscribers, when a new one is added as a consequence of a change in the price, the conceptual curve of demand shifts to the left or to the right. This depends if the price respectively increases or decreases. If we call Q=Q(p,N) the conceptual curve of demand, then the incremental consumer surplus that equals the externality effect described above, when the number of subscribers increases from Q (p, N) to Q (p, N + ∂N / ∂p) is the following (Mitchell & Vogelsang 1991):

$$\frac{\partial EX}{\partial P} = \int_p^\infty Q(p', N)dp' - \int_p^\infty Q(p', N + \frac{\partial N}{\partial p})dp' \quad (1)$$

In equation 1, EX represents the externality effect, that is the difference of consumer surpluses when there is a change in the number N of subscribers. EX will be used in the formulas of the model described next.

Optimal Model for Pricing Interconnection

The following model for optimal interconnection pricing is based on the model developed by Armstrong, Doyle & Vickers (1996) (called from now on the ADV model) to study the efficient component pricing rule (ECPR). The ADV model assume, in its basic form, the following conditions: contestability, the incumbent and the competitive fringe producing homogeneous goods, fixed production coefficients, and incapacity of bypassing the incumbent's essential facility.

The reasons for the choice of the ADV model are mainly three. First, it allows modeling a market with an incumbent owning the essential facility, and a competitive fringe, which needs that facility to access the market. Secondly, the maximizing welfare function permits including the externality function for obtaining the aggregate welfare of producers and consumers. And finally, it assumes contestability, making the model more realistic.

ADV Model without Externalities

The simplest case of the ADV model considers an industry with two firms: a dominant firm and an entrant firm (or competitive fringe). The dominant firm monopolizes the essential facility and the entrant must buy access to the incumbent in order to get to the consumers (one-way access).

The dominant firm has the cost function *C(q, z)*, where *q* is the number of units of final product supplied to consumers, and *z* is the number of units of access supplied to the entrant. Their marginal costs are respectively C_1 and C_2.

The entrant incurs also in cost *c(s)* for supplying *s* units of final product, and needs one unit of access for each unit of final product, paying *a* for each one. The dominant has a price of *P* for final products; the entrant charges *p* for each unit of final product. Therefore, the entrant has a margin of *m=p-a*. The consumer demand for the final product is represented by the function *X(P)*. The utility functions of the dominant and the entrant are stated in equation 2 and 3, respectively.

$$\Pi(P,m) \equiv PX(P) - ms(m) - C(X(P)) - s(m), s(m)) \quad (2)$$

$$\pi(m) \equiv max_{s\geq 0} : ms - c(s) \quad (3)$$

Thus, considering the sum of consumer surplus and firms' profits, a measure of the total social welfare, it follows that:

$$W(P,m) \equiv \nu(P) + \pi(m) + \Pi(P,m) \quad (4)$$

In equation 4, *v(P)* is the consumer surplus, and Π(P,m) and π*(m)* are the utility functions of the dominant and the entrant, respectively.

Following the work by Armstrong, Doyle & Vickers (1996), succinctly described above, the results for this basic model are equations 5 and 6 for a first-best solution,

$$P = C_1 \quad (5)$$

$$a = C_2 \quad (6)$$

and equations 7 and 8 for a second-best or Ramsey solution.

$$P = \frac{C_1}{1-\frac{\theta}{\eta_x}} \quad (7)$$

$$a = \frac{C_1}{1-\frac{\theta}{\eta_x}} - \frac{C_1 - C_2}{1-\frac{\theta}{\eta_s}} \quad (8)$$

In equations 7 and 8, η_x and η_s are the elasticity of demand for the final product, and the entrant's elasticity of supply with respect to the margin m. And $\theta = \lambda/(1+\lambda)$, with λ as the multiplier for the constraint of break-even for the dominant firm.

ADV Model with Externalities

In the case of a low penetration rate, network externalities are relevant when considering the setting of access charges, as it was already discussed. Therefore, this section proposes a modification of the ADV model seen in the previous subsection, adding to the demand function *X(P)* the component N(P) to consider the network externalities (it means that the number of consumers connected to the network depends on the price *P*). Hence the demand function would result in *X(P,N(P))*; in this way, the demand not only depends on price P, but also on the number of subscribers. The dominant firm's utility function appears in equation 9.

$$\Pi(P,N,m) \equiv$$
$$PX(P,N) - ms(m) - C(X(P) - s(m), s(m)) \quad (9)$$

The welfare function, the sum of the consumer surplus and the firms' utility, has to be changed also. It is necessary to add the externality effect (EX), matching the difference of consumer surpluses when there is a change in the number *N* of subscribers. The formula for *EX* was stated in equation 1. Equation 10 represents the new welfare function.

$$W(P,N,m) \equiv$$
$$\nu(P) + EX + \pi(m) + \Pi(P,N,m) \quad (10)$$

The maximization of equation 10 leads to the following values for the price *P* and the access charge *a*:

$$P = C_1 - \frac{\frac{\partial EX}{\partial P}}{\frac{\partial X}{\partial P} + \frac{\partial X}{\partial N}\frac{\partial N}{\partial P}} \quad (11)$$

$$a = C_2 - \frac{\frac{\partial EX}{\partial P}}{\frac{\partial X}{\partial P} + \frac{\partial X}{\partial N}\frac{\partial N}{\partial P}} \quad (12)$$

If the break-even constraint is active, that is the multiplier of the constraint $\lambda \geq 0$, then the results are:

$$P = C_1 - \frac{\lambda X(P,N) + \frac{\partial EX}{\partial P}}{(1+\lambda)(\frac{\partial X}{\partial P} + \frac{\partial X}{\partial N}\frac{\partial N}{\partial P})} \quad (13)$$

$$a = P - \frac{C_1 - C_2}{1-\frac{\theta}{\eta_s}} \quad (14)$$

Analysis of the Results

First of all, the solutions described above exist, as scale economies are present in the industry. On one hand, if scale economies exist, the average cost is decreasing and the cost function is convex. On the other hand, the objective function is concave, because the cost function is included with negative sign. This implies a maximum and a solution for the problem. When the beak-even constraint is active, it is necessary that the objective function is differentiable and concave, and the restriction is differentiable and convex, both in the non-negative octant, in order to apply the Kuhn-Tucker theorem (Chiang 1974). For the latter, as the condition implies cost minus income minor or equal to zero, the function becomes convex. So, it is possible to affirm that a solution for the model exist.

First-Best Pricing Alternative

In the case of the first-best solutions of the ADV model, when there are no externalities, the solutions are the marginal costs (equations 5 and 6). However when network externalities are present in the model, the values for *P* and *a* (equations 7 and 8) are the marginal costs corrected by an externality factor (EF):

$$EF = \frac{\frac{\partial EX}{\partial P}}{\frac{\partial X}{\partial P} + \frac{\partial X}{\partial N}\frac{\partial N}{\partial P}} \quad (15)$$

The externality factor decreases the price for the final product and the access charge. It is easy to verify that, on the one hand, if price *P* decreases, then the externality effect (EX) will be positive and $\partial EX/\partial P$ will be negative. On the other hand, if price *P* decreases, $\partial X/\partial P$ and $\partial N/\partial P$ will be negative, meanwhile $\partial X/\partial N$ will be positive.

Second-Best Pricing Alternative

The regulator, when applying the second-best pricing policy, tries to get the greatest consumer surplus allowing the firms with scale economies to break-even. Second-best or Ramsey prices accomplish the condition pointed in equation 16 (Train 1991).

$$\frac{P_1 - MC_1}{P_1}\epsilon_1 = \frac{P_2 - MC_2}{P_2}\epsilon_2 \quad (16)$$

The equation above relates the marginal costs of two goods (MC_1 and MC_2), with their prices (P_1 and P_2), and the price elasticities of demand (ϵ_1 and ϵ_2). For the analysis, we compare the first order conditions for the ADV model without externalities (equation 17) against the ADV model with externalities (equation 18). Both correspond to the form of equation 16.

$$\frac{P - C_1}{P}\eta_x \quad = \quad \frac{m - (C_1 - C_2)}{m}(-\eta_s) \quad (17)$$

$$\frac{P - C_1}{P} \frac{\frac{\partial X}{\partial P} + \frac{\partial X}{\partial N}\frac{\partial N}{\partial P}}{-\frac{X(P,N)}{P} - \frac{\partial EX}{\partial P}\frac{1}{\lambda P}} \quad =$$

$$\frac{m - (C_1 - C_2)}{m}(-\eta_s) \quad (18)$$

Comparing equations 17 and 18, the difference is in the price elasticity of the demand for the final product of the dominant firm. No change occurs in the side of the entrant firm.

Rewriting equation 18, we obtain:

$$\frac{P - C_1}{P} \frac{\frac{\partial X}{\partial P} + \frac{\partial X}{\partial N}\frac{\partial N}{\partial P}}{-\frac{X(P,N)}{P} - \frac{\partial EX}{\partial P}\frac{1}{\lambda P}} \quad =$$

$$\frac{m - (C_1 - C_2)}{m}(-\eta_s) \quad (19)$$

From the previous analysis we got that $\partial EX/\partial P$ is negative. In addition, P is greater than zero, and λ should be positive for the problem to have a solution according to the Kuhn-Tucker conditions. Therefore, the price elasticity of demand for dominant firm's final good decreases when the network externalities of the good increases, and vice versa.

Conclusions

One of the main issues for a regulator in the telecommunication industry is setting access charges. Traditionally, models in the literature have not included network externalities in their analysis, even though they are important when there is a low penetration rate. The model presented in this work, based in a model by Armstrong, Doyle & Vickers (1996), incorporate network externalities and allows comparing this result with the case without network externality. The model is developed considering a first and a second best pricing alternative.

The access charges are always lower when there is network externality in the market, i.e. when there is a low penetration rate. In the case of the Ramsey solution, the elasticity price of demand for the good provided by the dominant firm fall when the externality rises.

Given the lower prices in the presence of network externalities, the universal service is promoted, even though not necessarily in the most efficient way because of the deviations from the marginal costs.

Finally, if the model is considered in a scenario of implanting new technologies with scale economies and network externalities, it is important to note that a pricing model which do not consider those externalities could harm consumers. If the prices are calculated including network externalities, this positive effect will be transferred to consumers. Because of this, the regulator should generate the incentives for the firm to make the net grow.

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